Numerical models of Titan's internal structure without the liquid ocean

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In this paper models of the internal structure of Titan have been constructed. Conditions for deep water ocean existence depending on the heat flow values through the icy crust of satellite were discussed. It was shown that the internal ocean in Titan can’t be formed at the heat flow less 3.3 mW/m². At the first stage of this work the possible models of Titan without internal ocean has been built. According to model calculations, it turns out that Titan is a partially differentiated satellite, which in general involves the outer ice shell, rock-ice mantle and inner rock-iron core. The maximum size of the core is 1500 km with a density of rock-iron material 3.62 g/cm³. Maximum thickness of the external ice shell of Titan reaches 520 km. The total content of H₂O in satellite lies in the range of 45–51 wt.% depending on the rock-iron mantle component density, taken as 3.15–3.62 g/cm³ respectively.

Key words: satellites of giant planets, Titan, heat flow, the internal structure

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While studying of Titan, one of the most important key point is understanding history of appearance and evolution of this large icy satellite of Solar System. These studies are associated with attempts to understand the special feature of internal structure of Titan, characteristics of phase and chemical composition of its material.

Basics knowledge’s of Titan were obtained during the Cassini-Huygens spacecraft mission aimed at studying the Saturn System. The most important results of this expedition are consists on obtained geophysical data on the mass, mean density and moment of inertia of the satellite.

Analysis of these data allows us to conclude that Titan in many parameters is similar to the Jovian satellite Callisto, whose internal structure discussed in detail in [Kronrod and Kuskov, 2003; Kronrod and Kuskov, 2005]. According to these studies, Callisto is partially differentiated satellite, in the depths of which an internal ocean of liquid water up to 150-170 km can possible exists. In respect to the Titan, analysis of all available information does not allow to say definitely whether there are (or not) in its depths some reservoirs of liquid water.

The existence a liquid ocean in satellite interiors depends on the heat transfer processes and temperature distribution over its depth. Possible variants of the radial temperature distribution in Titan, corresponding to different values of heat flow through the satellite’s icy crust, shown schematically in Figure 1. The temperature profile of Titan's inner regions was set with the assumption of conductive (in region of low temperatures and pressures at the satellite surface) and convective adiabatic medium typical for the deeper layers of Titan.

As can be seen from the figure, each value of the heat flow corresponds to a specific structure of water-ice shell of Titan (Figure 1 represents a different Titan’s models: without internal ocean and with the ocean of intermediate and largest depth). By this means, choice of temperature profile dictates the phase composition (and hence the density distribution) in the water-ice shell of the satellite, and also determines the maximum thickness of its upper icy crust, composed by ice Ih, and the depth of its subcrustal ocean.

In accordance with literature data [Hussmann et al., 2006], the heat flux of Titan, defined by its radiogenic heat only, may be equal to 1.18 mW/m². Consideration of additional tidal heating gives the value of the heat flow 7.1 mW/m² [Mitri and Showman, 2008]. In present paper on the base of heat transfer equations and using the water phase diagram all possible values of Titan’s heat fluxes have been calculated. Theirs application to Titan leads to variety models of satellite's water-ice shell structure (Fig. 2).
From the Fig. 2 follows that at heat flow less than 3.3 mW/m², Titan’s internal ocean is not formed. In this case the satellite’s outer water-ice shell is represented only by water ice (thickness of the subcrustal ocean (Hw) is equal to zero), whereas under the heat flow equal 7.1 mW/m², the outer ice crust up to 80 km and a subcrustal ocean of liquid water up to 315 km can be formed.

In context of this work, the possible models of Titan's internal structure without inner ocean, which correspond to heat flow 3.3 mW/m², have been considered. In general case the three-layer model of Titan was constructed. This model includes an outer icy shell, inner rock-iron core and the rock-ice mantle located between them. The special (extreme) cases of this model serves the two-layer models of Titan, including the thick rock-ice layer in combination with either the inner rock-iron core (without an external ice crust), or with an external ice crust, but without the rock core.

Fig. 1. Phase diagram of water and the distribution of temperature in the Titan’s icy crust. Straight lines - conductive temperature profiles through the external (ice-Ih) crust. The dashed lines – adiabatic convective heat transfer in the water subcrustal ocean and in high-pressure ice medium.

H, H_Ih - the distance from the satellite's surface (depth) and the thickness of the external ice-Ih crust.
To construct the models of Titan’s internal structure the following background information was used:

- The physical characteristics of the satellite (T, P, gravity acceleration, radius, density, mass, moment of inertia). This parameters form the main restrictions for the developed model.
- Geochemical data on density and composition of meteoritic matter (L/LL chondrites). It is suggested that the composition of Titan rock-iron material is corresponds precisely to this matter.
- Thermodynamic equations of state for water and high pressure ices.
- In compliance with mathematical approach taken in this paper, a model of the internal structure of Titan described by the system of following equations [Kronrod and Kuskov, 2003]:
  
  1. Equations of hydrostatic equilibrium:
     \[
     \frac{dP}{dR} = -\rho(R) \cdot g(R); \quad \frac{dg}{dR} = -4\pi \cdot G \cdot \rho(R) - 2g(R)/R, \tag{1}
     \]
  
  2. The equations of the satellite mass and moment of inertia:
     \[
     I = \frac{8}{15} \pi \sum_{i=0}^{n} \rho_i \left( R_i^5 - R_{i+1}^5 \right); \quad M = \frac{4}{3} \pi \sum_{i=0}^{n} \rho_i \left( R_i^3 - R_{i+1}^3 \right), \tag{2}
     \]
  
  3. The equation for calculating ice component concentration in mantle: 
     \[
     C_{\text{ice}} = \frac{\rho_{\text{ice,m}} (\rho_{\text{Fe-Si}} - \rho_m)}{\rho_m (\rho_{\text{Fe-Si}} - \rho_{\text{ice,m}})}, \tag{3}
     \]

where \( P, R, M, I \) - pressure, radius, mass and moment of inertia of Titan, \( \rho(R) \) - the density of water-ice shell, \( g(R) \) - gravity acceleration, \( R_i, R_{i+1} \) - are the greatest and least radii of the \( i \)th layer with the density \( \rho_i \), \( \rho_{\text{ice,m}} \) - the ice’s average density in mantle, \( \rho_m \) - mantle average density, \( \rho_{\text{Fe-Si}} \) - density of the rock–iron component, \( C_{\text{ice}} \) - mass fraction of ices in the ice–rock mantle.

The system of equations (1-3) is numerically integrated in the range of depths from the surface to the calculated depth of the satellite. In the calculations it was assumed that Titan’s rock-iron substance has a constant density with depth. The density of the inner ice-free core was assumed to be 3.62 g/cm\(^3\) in all calculations. Density of Fe-Si component in the rock-ice layer of Titan was chosen in the range typical for the ordinary L/LL chondrites with taking into account the silicates hydration: from 3.15 to 3.62 g/cm\(^3\).

Calculations performed by two-layer models of Titan allowed to estimate the maximum size of its outer ice crust and the inner rock-iron core. It was shown that the maximum size of Titan’s core achieved through a two-layer model without an external ice shell (Fig. 3a). In this case, partially differentiated Titan is formed. The satellite was represented as inner rocky core with radius of about 1500 km and the outer thick layer of rock-ice material with average density about of 1.4 g/cm\(^3\).

In case when Titan has significant large outer icy crust, its inner rock-iron core not forms, and the interior of the satellite is represented by a homogeneous substance of average density 2.6 g/cm\(^3\), which is likely refer to a mixture of rock material and the high-pressure ice. This structure corresponds to the two-layer model of partially differentiated Titan without inner core (Fig. 3b). On this model the maximum possible thickness of an ice crust reaches 515-520 km at an average density of 1.15 g/cm\(^3\). The mass of water contained in outer ice shell is about 30% of the total mass of Titan.

The phase composition of Titan's icy crust is represented by water ice I\( \text{h} \), III, V, VI, which consistently replacing each other in depth. The upper conductive layer of the icy I\( \text{h} \)-crust does not exceed 160 km and has an average density of about 0.94 g/cm\(^3\).

The density of ice in the rock-ice Titan interior varies according to the equations of state and reaches values of 1.77 g/cm\(^3\) at pressures in the center of the satellite equal to 48 kbar (the region of ice VII existence).

A generalized three-layer model of Titan, which differentiated on the icy shell, rock-ice mantle, and rock-iron core, is shown in Fig. 4. In this model many varieties of the internal structure of the satellite are realized. All models differ in the thickness of outer ice shell, and concurrently with it the size of the inner rock core, as well as the size and density of the rock-ice mantle are also changing.

The total content of ice in Titan was determined by its summary concentration in the outer ice shell and in the rock-ice mantle. The calculations have shown that the bulk amount of ice in the satellite is
strongly depended on the density of the mantle rock component and less on extent on the thickness of outer water-ice shell.

In the two-layer models of Titan with a minimal thickness of an ice crust and the density of Fe-Si component 3.15 - 3.62 g/cm³, the mass fraction of ice in the satellite is 49.5 - 50.6 wt. %, respectively. At the maximum size of an ice shell allowable range of the ice content in Titan increases, respectively, to 45.1–51.1 wt. %.

Thus, for a whole range of Titan possible models the total water content in satellite is placed in the range from 45 to 51 wt. %.

Figure 3. Two-layer models of Titan:

3a - a model with a maximum rock-iron core,
3b - a model with a maximum ice crust. Gray lines show the change in ice density in the ice crust and in the rock-ice region.

Figure 4. A general model of Titan's internal structure without subcrustal ocean.

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References
